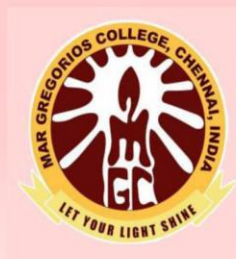# MAR GREGORIOS COLLEGE

OF ARTS & SCIENCE

Block No.8, College Road, Mogappair West, Chennai – 37

Affiliated to the University of Madras
Approved by the Government of Tamil Nadu
An ISO 9001:2015 Certified Institution

DEPARTMENT OF COMMERCE

SUBJECT NAME: ELEMENTS OF OPERATIONS RESEARCH

SUBJECT CODE: CDZ4A

SEMESTER: IV

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ELEMENTS OF OPERATIONS RESEARCH

SYLLABUS

OBJECTIVES
- To Facilitate this Understanding of the Concept of Operations Research
- To Help the Students to Understand the Various Techniques of Solving Problems

OUT COME:
- Understanding of the Concept of Operations Research and to Help the Students toUnderstand the Various Techniques of Solving Problems

UNIT I : Introduction
Operations Research- Meaning-Definition - Origin and History- Characteristic Features – Need-Scope –Steps- Techniques- Application- Limitations

UNIT II : Linear Programming Problem Lpp
Meaning- Requirements- Assumptions- Applications- Formulating Lpp –Advantages- Limitations Formulating LP Model (Simple Problems Only)

UNIT III: Methods Of Lpp
Obtaining Optimal Solution for Linear Programming Problem (LPP)-Graphical Method - Problems --Simplex Method for Type of LPP and for Slack Variable Case - Maximization Function -Minimization Function (Simple Problem Only)

UNIT IV : Transportation Problems
Meaning –(Initial Basic Feasible Solution)Assumptions -Degenerate Solution -North - West Corner Method- Least Cost Method -Vogels Approximation Method - Assignment Problems- Features -Transportation Problem Vs Assignment Problem - Hungarian Method (Simple Problems Only)

UNIT V: Game Theory
Meaning- Types of Games- Basic Assumptions- Finding Value of Game for Pure Strategy - Mixed Strategy -Indeterminate Matrix and Average Method -Graphical Method -Pure Strategy- Saddle Point Payoff Matrix Value of Game (Simple Problems Only)

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UNIT I
Introduction

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Operations research meaning

Operationss research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In **operationss research**, problems are broken down into basic components and then solved in **defined** steps by mathematical analysis.

Definition of Operations Research.

It is the application of scientific methods, techniques and tools to problems involving the operationss of a system so as to provide those in the control of the system with optimum solutions to the problems.

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Characteristics (Features) of Operations Research:
Main characteristics of operationss research (O.R.) are follows:

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(i) Inter-Disciplinary Team Approach:

This requires an inter-disciplinary team including individuals with skills in mathematics, statistics, economics, engineering, material sciences, computer etc.

(ii) Wholistic Approach to the System:

While evaluating any decision, the important interactions and their impact on the whole organisation against the functions originally involved are reviewed.

(iii) Methodological Approach:

O.R. utilises the scientific method to solve the problem

(iv) Objective Approach:

O.R. attempts to find the best or optimal solution to the problem under consideration, taking into account the goals of the organisation.

Scope of Operations Research

1. Finance, Budgeting and Investment:
i. Cash flow analysis, long range capital requirement, investment portfolios, dividend policies,
ii. Claim procedure, and
iii. Credit policies.

2. Marketing:
i. Product selection, competitive actions,
ii. Number of salesmen, frequencies of calling on, and
iii. Advertising strategies with respect to cost and time.

3. Purchasing:
i. Buying policies, varying prices,
ii. Determination of quantities and timing of purchases,
iii. Bidding policies,
iv. Replacement policies, and
v. Exploitation of new material resources.

4. Production Management:
i. Physical distribution: Location and size of warehouses, distribution centres and retail outlets, distribution policies.
ii. Facilities Planning: Number and location of factories, warehouses etc. Loading and unloading facilities.
iii. Manufacturing: Production scheduling and sequencing stabilisation of production, employment, layoffs, and optimum product mix.
iv. Maintenance policies, crew size.
v. Project scheduling and allocation of resources.

5. Personnel Management:
i. Mixes of age and skills,
ii. Recruiting policies, and
iii. Job assignments.

6. Research and Development:
i. Areas of concentration for R&D.
ii. Reliability and alternate decisions.
iii. Determination of time-cost trade off and control of development projects.

Limitations of Operations Research:

i. These do not take into account qualitative and emotional factors.
ii. These are applicable to only specific categories of decision-making problems.
iii. These are required to be interpreted correctly.
iv. Due to conventional thinking, changes face lot of resistance from workers and sometimes even from employer.
v. Models are only idealised representation of reality and not be regarded as absolute.

Techniques of Operations Research

Techniques of Operations Research:

Important techniques of Operations Research are being described hereunder:

(i) Inventory Control Models:

Operations Research study involves balancing inventory costs against one or more of

the following costs:
i. Shortage costs.
ii. Ordering costs.
iii. Storage costs.

iv. Interest costs.

This study helps in taking decisions about:
i. How much to purchase.
ii. When to order.
iii. Whether to manufacture or to purchase i.e., make and buy decisions.
The most well-known use is in the form of Economic Order Quantity equation for finding economic lot size.

(ii) Waiting Line Models:

These models are used for minimising the waiting time and idle time together with the costs associated therewith.

Waiting line models are of two types:

(a) Queuing theory, which is applicable for determining the number of service facilities and/or the timing of arrivals for servicing.

(b) Sequencing theory which is applicable for determining the sequence of the servicing.

(iii) Replacement Models:

These models are used for determining the time of replacement or maintenance of item, which may either:

(i) Become obsolete, or
(ii) Become inefficient for use, and
(iii) Become beyond economical to repair or maintain.

(iv) Allocation Models:

These models are used to solve the problems arising when:

(a) There are number of activities which are to be performed and there are number of alternative ways of doing them,

(b) The resources or facilities are limited, which do not allow each activity to be performed in best possible way. Thus these models help to combine activities and available resources so as to optimise and get a solution to obtain an overall effectiveness.

(v) Competitive Strategies:

Such type of strategies are adopted where, efficiency of decision of one agency is dependent on the decision of another agency. Examples of such strategies are game of cards or chess, fixing of prices in a competitive market where these strategies are termed as "theory".

(vi) Linear Programming Technique:
These techniques are used for solving operations problems having many variables subject to certain restrictions. In such problems, objectives are—profit, costs, quantities manufactured etc. whereas restrictions may be e.g. policies of government, capacity of the plant, demand of the product, availability of raw materials, water or power and storage capacity etc.

(vii) Sequencing Models:
These are concerned with the selection of an appropriate sequence of performing a series of jobs to be done on a service facility or machine so as to optimise some efficiency measure of performance of the system.

(viii) Simulation Models:
Simulation is an experimental method used to study behaviour over time.

(ix) Network Models:
This is an approach to planning, scheduling and controlling complex projects.

Applications of Operations Research:

These techniques are applied to a very wide range of problems.

Here only some of the common applications are being mentioned:
(i) Distribution or Transportation Problems:
In such problems, various centres with their demands are given and various warehouses with their stock positions are also known, then by using linear programming technique, we can find out most economical distribution of the products to various centres from various warehouses.

(ii) Product Mix:
These techniques can be applied to determine best mix of the products for a plant with available resources, so as to get maximum profit or minimum cost of production.

(iii) Production Planning:
These techniques can also be applied to allocate various jobs to different machines so as to get maximum profit or to maximise production or to minimise total production time.

(iv) Assignment of Personnel:
Similarly, this technique can be applied for assignment of different personnel with different aptitude to different jobs so as to complete the task within a minimum time.

(v) Agricultural Production:

We can also apply this technique to maximise cultivator's profit, involving cultivation of number of items with different returns and cropping time in different type of lands having variable fertility.

(vi) Financial Applications:

Many financial decision making problems can be solved by using linear programming technique.

Some of them are:

(i)   To select best portfolio in order to maximise return on investment out of alternative investment opportunities like bonds, stocks etc. Such problems are generally faced by the managers of mutual funds, banks and insurance companies

(ii) In deciding financial mix strategies, involving the selection of means for financing firm, projects, inventories etc.

History of operation research

Operations Research (Operational Research, O.R., or Management science)includes a great deal of problem-solving techniques like Mathematical models, Statistics and algorithms to aid in decision-making. O.R. is employed to analyze complex real-world systems, generally with the objective of improving or optimizing performance.

In other words, Operations Research is an interdisciplinary branch of applied mathematics and formal science which makes use of methods like mathematical modeling, algorithms statistics and statistics to reach optimal or near optimal solutions to complex situations.

It is usually worried about optimizing the maxima (for instance, profit, assembly line performance, bandwidth, etc) or minima (for instance, loss, risk, cost, etc.) of some objective function. Operational Research aids the management to accomplish its objectives utilizing scientific methods.

Based on the **history of Operations Research**, it is believed that Charles Babbage (1791-1871) is the father of Operational Research due to the fact that his research into the cost of transportation and sorting of mail resulted in England's universal Penny Post in 1840.

Thomas Edison made use of Operational Research, contributing in the antisubmarine war, with his greats ideas, like shields against torpedo for the ships.

In 1947 -Janos Von Neumann published his work called "Theory of Games", that provided the basics Mathematicians to Linear Programming.At a later time, in 1947, he viewed the similitude among them Programming linear problems and the matrix theory that developed himself.

In 1939, mathematical Russian L. Kantorovich, in association with the mathematical Dutchman T. Koopmans, developed the mathematical theory called "Linear programming", thanks to that went rewarded with the Nobel.

In 1840., It is believed that Charles Babbage is the father of the Operational Research due to his research about the transportation's costs and sorting of mail realized for the Uniform Penny Post in in England

1941 and 1942, Kantorovich and Koopmans studied in independent ways the Transport Problem for first time, knowing this type of problems like problem of Koopmans-Kantorovich. For his solution, they used geometric methods that are related to Minkowski's theorem of convexity.

In 1947,To notice the range of this new discipline, England created another groups of the same nature in order to obtain optimal results in the dispute. Just like United States (USA), when joined the War in 1942, creating the project SCOOP (Scientific Computation Of Optimum Programs), where was working George Bernard Dantzig, who developed in 1947 the Simplex algorithm.

June 25, 1948.During the Cold War, the old Soviet Union (URRS), excluded of the Plan Marshall, wanted to control the terrestrial communications, including routes fluvial, from Berlin. In order to avoid the rendition of the city, and his submission to be a part of the deutsche communist zone, England and United States decided supplying the city, or else by means of escorted convoys (that would be able to give rise to new confrontations) or by means of airlift, breaking or avoiding in any event the blockage from Berlin. Second option was chosen, starting the Luftbrücke (airlift) at June 25, 1948.

May 12, 1949. This went another from the problems in which worked the SCOOP group, in December of that same year, could carry 4500 daily tons, and after studies of Research Operations optimized the supplying to get to the 8000~9000 daily tons

in March of 1949. This cipher was the same that would have been transported for terrestrial means, for that the Soviet decided to suspend the blockage at May 12, 1949.

After second World War, the order of United States' resources (USA) (energy, armaments, and all kind of supplies) took opportune to accomplish it by models of optimization, resolved intervening linear programming.

In 1952, The first result of these techniques was given at the year 1952, when a SEAC computer from was used National Bureau of Standards in way to obtain the problem´s solution. The success at the resolution time was so encouraging that was immediately used for all kind of military problems, like determining the optimal height which should fly the planes to locate the enemy submarines, monetary founds management for logistics and armament, including to determine the depth which should send the charges to reach the enemy submarines in order to cause the biggest number of casualties, that was translated in a increase in five times in Air Force's efficacy.

During the 50's and 60's decade, grows the interest and developing of Operational Research, due to its application in the space of commerce and the industry. Take for example, the the problem of the calculation of the optimal transporting plan of sand of construction to the works of edification of the city of Moscow, which had 10 origins points and 230 destiny. To resolve it, was used and Strena computer, that took 10 days in the month of june of 1958, and such solution contributed a reduction of the 11 % of the expenses in relation to original costs.

Modes of OR

Types of Model with examples and say the simplified ways for the best model.

Operations Research Models:
Operations Research model is an idealised representation of the real life situation and represents one or more aspects of reality. Examples of operations research models are: a map, activity charts balance sheets, PERT network, break-even equation, economic ordering quantity equation etc. Objective of the model is to provide a means for analysing the behaviour of the system for improving its performance.

Classification of Models:

Models can be classified on the basis of following factors:

1. By degree of Abstraction:
i. Mathematical models.
ii. Language models.
2. By Function:
i. Descriptive models.
ii. Predictive models.
iii. Normative models for repetitive problems.
3. By Structure:
i. Physical models.
ii. Analogue (graphical) models.
iii. Symbolic or mathematical models.

4. By Nature of Environment:

i. Deterministic models.
ii. Probabilistic models.
5. By the Time Horizon:
i. Static models.
ii. Dynamic models.

<u>**Characteristics of a Good Model:**</u>
i. Assumptions should be simple and few.
ii. Variables should be as less as possible.
iii. It should be able to assimilate the system environmental changes without change in its framework.
iv. It should be easy to construct.
Constructing the Model:
A mathematical model is a set of equations in which the system or problem is described. The equations represent objective function and constraints. Objective function is a mathematical expressions of objectives (cost or profit of the operations), while constraints are mathematical expressions of the limitations on the fulfilment of the objectives.
These expressions consist of controllable and uncontrollable variables.
Some of the common simplifications are:
i. Omitting certain variables.
ii. Aggregating (or grouping) variables.
iii. Changing the nature of variables e.g., considering variables as constant or continuous.
iv. Changing relationship between variables i.e., considering them as linear or straight line.
v. Modify constraints.

UNIT II

Linear Programming Problem (LPP)

Linear programming Problem (LPP)

Linear programming is an optimization technique for a system of linear constraints and a linear objective function. An **objective function** defines the quantity to be optimized, and the goal of linear programming is to find the values of the variables that maximize or minimize the objective function.

This kind of problem is perfect to use linear programming techniques on.

- All of the quantifiable relationships in the problem are linear.
- The values of variables are constrained in some way.
- The goal is to find values of the variables that will maximize some quantity.

Meaning of graphical method

Graphical method of linear programming is used to solve problems by finding the highest or lowest point of intersection between the objective function line and the feasible regionon a graph.

LPP Assumptions

There are several assumptions on which the linear programming works, these are:

1. **Proportionality:** The basic assumption underlying the linear programming is that any change in the constraint inequalities will have the proportional change in the objective function.

2. **Additivity:** The assumption of additivity asserts that the total profit of the objective function is determined by the sum of profit contributed by each product separately.
3. **Continuity:** Another assumption of linear programming is that the decision variables are continuous. This means a combination of outputs can be used with the fractional values along with the integer values.
4. **Certainty:** Another underlying assumption of linear programming is a certainty, i.e. the parameters of objective function coefficients and the coefficients of constraint inequalities is known with certainty.
5. **Finite Choices:** This assumption implies that the decision maker has certain choices, and the decision variables assume non-negative values. The non-negative assumption is true in the sense, the output in the production problem can not be negative.

Advantages and limitations (or)disadvantages:

LP has been considered an important tool due to following reasons:

1. LP makes logical thinking and provides better insight into business problems.
2. Manager can select the best solution with the help of LP by evaluating the cost and profit of various alternatives.
3. LP provides an information base for optimum allocation of scarce resources.
4. LP assists in making adjustments according to changing conditions.
5. LP helps in solving multi-dimensional problems.

Limitations (or) disadvantages:

1. This technique could not solve the problems in which variables cannot be stated quantitatively.
2. In some cases, the results of LP give a confusing and misleading picture.
3. LP technique cannot solve the business problems of non-linear nature.
4. The factor of uncertainty is not considered in this technique.
5. This technique is highly mathematical and complicated.

<u>Simple steps for Graphical method</u>

Step 1: Set the Objective function   i.e. Maximisation of profit and Minimisation of expenses

Example $Z = 12X_1 + 16X_2$

Step 2: Convert Inequality into **Equality**   by equation

$10X_1 + 20X_2 = 120$

$8X_1 + 8X_2 = 80$

Solve the equation taking when X1 is 0;   what is X 2 ?

X_1	0	?
X_2	?	0

Step 3: Plot both equation on graph sheet

Step 4: Find the feasible region which is found under the two equations for Maximisation Problems like the Region found above two equation are called Feasible region in case of Minimisation Problems.

Step 5: Find all the corner values

Step 6: Apply all the Corner values in the Objective Function ie $Z = 12X_1 + 16X_2$

Which is **Maximum** is the solution for **Maximisation Problem** and

Which is **Minimum** is the solution for **Minimisation Problem** and

UNIT III
Methods of LPP

Simplex Method

The **Simplex Method or Simplex Algorithm** is used for calculating the optimal solution to the linear programming problem. In other words, the simplex algorithm is an iterative procedure carried systematically to determine the optimal solution from the set of feasible solutions.

Why introduce slack variable

slack variable is a **variable** that is added to an inequality constraint to transform it into an equality. **Introducing** a **slack variable**replaces an inequality constraint with an equality constraint and a non-negativity constraint on the **slack variable**.

feasible solution meaning

A **feasible solution** is a set of values for the decision variables that satisfies all of the constraints in an optimization problem. The set of all **feasible solutions** defines the **feasible** region of the problem.

basic feasible solution

A **basic solution** that satisfies all the constraints defining or in other words, one that lies within is called a **basic feasible solution**. let Ax=b the system of the 'm' equation with 'n' unknown variables.

LIMITATIONS OF LPP SIMPLEX METHOD

1. Simplex method Involves understanding of many conceptual technical aspects. These cannot be understood by any manager not conversant with the subject.
2. Linear programming problems need lot of expertise, time and are cumbersome. A number of steps have to be adopted to proceed in a systematic manner before one can arrive at the solution.

3. Graphic solution method has lot of applications and is relatively short and simple. However, it has limitations and cannot be applied to problems with more than two variables in the objective function.
4. Simplex method of LPP can be applied to problems with more than two variables in the objective function, the procedure adopted is complicated and long.
5. LPP does not lead to 'a unique' optimal solution. It can provide different types of solutions like feasible solution, infeasible solution, unbounded solution, degenerate solution etc.

Simplex Method steps

Step 1: Set the Objective function with introduction S_1 and S_2 i.e. Maximisation of profit and Minimisation of expenses

Example $Z=12X_1 +16X_2 +0S_1+0S_2$

Step 2: Convert **Inequality** into **Equality** by equation

$10X_1 +20X_2 + S_1=120$

$8X_1+8X_2+ S_2=80$

Step 3: Construct a table using the above object function and inequality Constraint values

Cij	Cj	12	Key Column 16	0	0	Solution	Ratio	
	BV	X1	**X2** Incoming	S1	S2			
0	**S1 Out going**	10	20 Key Elemnt	1	0	120	$\frac{120}{20}$ = **6**	**Key Row**
0	S2	8	8	0	1	80	$\frac{80}{8}$ = 10	
	Zj	0	0	0	0	0		

12   **16**   0   0

Step 4: Find the Valuesof Zj

Zj=Cij x Bij   ie for X1 Column 0x10=0 + 0 x8 = 0

X2 Column 0x20=0 + 0 x8 = 0

Step 5:Find the Optimality

For Maximisation Problems Cj- Zj $\leq$ 0
For Minimisation Problem Cj- Zj $\geq$ 0
Step 6: If the Optimality is not come, go for Next Iteration Table by the following additional setps

a) bringing **incoming variable** by selecting the **Maximum Column Value** (Cj-Zj), that is **key column**
b) Find out the Ratio by = <u>**Solution**</u>
   Corresponding Column Value

c) Select the least Row Ratio , that Row is called **Key Row, that Row S Value is Out Going Variable**
d) **Find the Intersection of both the Row and Column, that Variable is called Key Element**
e) For the Next iteration find out new Value by the following
   1. in the **Key Row**, find new value by   =   <u>Key Row Value</u>

   Corresponding Key Column Value

In the **Non Key Row**.,

Find the New Value =Old Value - Corr.Key Row Value x Corr.Key Column Value

Key Element

Step 7 Repeat From the Step 3

UNIT IV

Transportation and Assignment Problems

ASSIGNMENT

Introduction

The assignment problem is a special case of transportation problem wherein the number of resources (origin) equals number of activities (destinations). The capacity and demand value is exactly one unit i.e. only one unit can be supplied from each origin and each destination also requires exactly only one Unit

OBJECTIVE ASSIGNMENT PROBLEM

The **objectives** alone are considered as fuzzy. The classical **assignment problem** refers to a special class of linear programming **problems**. Linear programming is one of the most widely used decision making tool for solving real world **problems**

In shortly in objective

Objectives : The objective is to determine which origin should supply specific units to which destination.

STEPS of assignment problem

1.   Deduct ROW MINIMUM from all the elements in each row for all rows.

2.   From such reduced matrix, deduct COLUMN MINIMUM from all elements in each column for all the columns.

3.   To find optimal solution,

Cover MAXIMUM NUMBER OF ZEROS by drawing MINIMUM NUMBER OF VERTICAL OR HORIZONTAL LINES which should be equal to order of matrix.

*** If the minimum number of lines are not equal to order of matrix (no optimal solution),
uncovered elements are reduced by smallest element in uncoverd area and intersection element is

added with such smallest element (no change in the covered area). Repeat the same until you are getting optimal solution.

4.   Make one assignment to least possible zero in each row and in each column. Subsequently, strike off other zeros found in the same row and same column.

*Least possible zero = giving preference to row or column which has minimum zero(s)

5.   Add original values of assignments located places to get the result.

Note:
1. Unbalanced problems : If the number of columns is not equal to number of rows, dummy column should be added with zero elements and vice versa.

2. Maximization case: If profit is given (usually cost is given), conversation of maximization problem into minimization problem by deduction of all elements in all rows and columns from the large value of the total elements. (Largest – All other smallest elements).

Hungarian method
The **Hungarian** **method** is a combinatorial optimization **algorithm** that solves the **assignment** problem in polynomial time and which anticipated later primal-dual**methods**.

What is Dummy activity?

a **dummy activity** is a simulated **activity** of sorts, one that is of a zero duration and is created for the sole purpose of demonstrating a specific relationship and path of action on the arrow diagramming method.

TRANSPORTATION

In the process of transportation of goods from one place to various distribution centers(origin) to various distribution centers(destination), transportation expenses are incurred. Some times it may be more due to random calculation. So in order to avoid increase of cost in transportation, the least cost is selected by applying a suitable method.

NORTH-WEST CORNER METHOD:

The **North-West Corner Rule** is a method adopted to compute the initial feasible solution of the transportation problem. The name North-west corner is given to this method because the basic variables are selected from the extreme left corner.

Least Cost Method

The **Least Cost Method** is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost.

The lower cost cells are chosen over the higher-cost cell with the objective to have the least

cost of transportation.

Vogel's Approximation Method

The **Vogel's Approximation Method** or **VAM** is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense.

NORTH-WEST CORNER METHOD STEPS:

1.   Begin from NORTH-WEST CORNER CELL(Upper Left Hand Corner) of the transportation table.

2.   ALLOT the respective Row Total or Column Total, whichever is less, in the North-West Cell .

*Side-by-side, corresponding to such row or column, mention the remaining balance to be allotted.

3.   Strike off the respective Row or Column as a sign of full allotment (which has remaining balance-zero) made.

4.   Select the next North-West Corner Cell and repeat the first three steps for remaining Rows and columns till possible allotments to be made.

Note: But, the total allotment made should be equal to m+n-1 in order to get feasible solution.

5.   Now, add all the values found by multiplying the transportation cost with allotment made to find the total transportation cost.

LEAST COST METHOD STEPS:

1. Choose the Least Cost Cell.

2. Allot the respective Row Total or Column Table, whichever is less, in the Least Cost Cell.

* Side-by-side, corresponding to such row or column, mention the remaining balance to be allotted.

3.   Strike off the respective Row or Column as a sign of full allotment (which has remaining balance-zero) made.
4.   Select the next Least Cost Cell and repeat the first three steps for remaining Rows and columns till possible allotments to be made.

Note: But, the total allotment made should be equal to m+n-1 in order to get feasible solution.

5.   Now, add all the values found by multiplying the transportation cost with allotment made to find the total transportation cost.

VOGEL's APPROXIMISATION METHOD (VAM) STEPS:

1. Find out the difference between two least cost in each column and in each row.

2. Select the maximum difference among them and locate the lowest cell corresponding to the maximum difference.

3. Allot the respective row total or column total whichever is less in such lowest cost cell.

4. Strike off the row or column or both as a sign of allotment fully made.

5. Repeat the first four steps for the remaining rows and columns till the possible allotments are made.

6. Now add the values found by multiplying the transportation cost with full allotment made to find out the totl transportation cost. *Generally, preference is given to minimum cost and possible maximum allotment.

MXIMISATION CASE:

If profit is given (usually cost is given), conversation of maximization problem into minimization problem by deduction of all elements in all rows and columns from the large value of the total elements. (Largest - All other smallest elements).

DEGENERACY:

In transportation problem, if total allotment is not equal to m+n-1, it is called degeneracy. In this case select the least unallotted (unallotted cell) and allot Epsilon (value close to zero or between 0 and 1) in such a least unallotted cell. The remaining steps are same as we followed earlier.

UNIT V
Game Theory

Game Theory

Introduction

Game theory was developed for decision making under conflicting situations where there are one or more opponents (players).

Definition:

It was developed by Prof. John Von Neumann and Oscar Morgenstern in 1928The games like chess, poker, etc. have the characteristics of competition and are played according to some definite rules. Game theory provides optimal solutions to such games, assuming that each of the players wants to maximize his profit or minimize his loss.

Meanings of strategy

Pure strategy - It is a decision, in advance of all plays, always to choose a particular course of action.

Mixed strategy - It is a decision, in advance of all plays, to choose a course of action for each play in accordance with some particular probability distribution.

Optimal strategy: The course of action which maximizes the profit of a player or minimizes his loss is called an optimal strategy.

Payoff: The outcome of playing a game is called payoff.

Payoff matrix: When the players select their particular strategies, the payoffs (gains or losses) can be represented in the form of a matrix called the payoff matrix.

Saddle point: A saddle point is an element of the payoff matrix, which is both the smallest element in its row and the largest element in its column. Furthermore, the saddle point is also regarded as an equilibrium point in the theory of games.

Value of the game: It refers to the expected outcome per play when players follow their optimal strategy.

GAME THEORY ASSUMPTIONS

1. There are finite numbers of competitors.
2. There is conflict of interests between them.
3. Each player has available with him finite courses of action.
4. Players know all possible available choices but do not know which one is going to be chosen.
5. Players simultaneously select their respective courses of action.
6. The payoff is fixed and determined in advance.

MERITS or SIGNIFICANCE or IMPORTANCE

1. **Helps in decision making**: Game theory develops a framework for analysing decision-makings under the situations of inter-dependence of firms with existing uncertainties about the competitor's reactions to any course of action adopted by a firm.

2. **Provide scientific quantitative technique**: This theory outlines a scientific quantitative technique which can be fruitfully used by players to arrive at an optimal strategy, given firm's objectives.

3. **Gives insight into situation of conflicting interests**: game theory gives insight into several less-known aspects which arise in situations of conflicting interests. For example, it describes and explains the phenomena of bargaining and coalition-formation.

Demerits or Limitations

1. **Unrealistic:** The assumption that the players have the knowledge about their own pay-offs and pay-offs of others is rather unrealistic. He can only make a guess of his own and his rivals' strategies.
2. **More Complication:** As the number of maximini and minimax show that the gaming strategies becomes increasingly complex and difficult. In practice, there are many firms in an oligopoly situation and game theory cannot be very helpful in such situation.
3. **Not Practical**: The assumptions of maximum and minimax show that the players are risk-averse and have complete knowledge the strategies. These do not seen practical.
4. **No Secrecy: Rather** than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out collusion. Thus, the mixed strategies are also not very useful.

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